**Chapter 1: Equations and Inequalities**

1.8 🡪 Other Types of Inequalities

To solve a polynomial inequality such as $ x^{2}-2x-3<0$ you can use the fact that a polynomial can change signs only at its zeros (x-values that make the polynomial = to zero).

Between two consecutive zeroes a polynomial must be entirely positive or entirely negative. These zeros are the **critical numbers** of the inequality, and the resulting intervals are the **test intervals** for the inequality.

$x^{2}-2x-3=(x+1)(x-3)$

The polynomial above has two zeros, $x=-1$ and $x=3$, and these zeros divide the real number line into three test intervals:

3

-1

$ \left(-\infty , -1\right), \left(-1, 3\right), and (3, \infty )$

-----------------------------------------------------------------------------------------------------------------------------

SOLVING A POLYNOMIAL INEQUALITY

1. $x^{2}-x-6<0$ Standard Form

 ( )( ) = 0 Factor

 Critical Numbers

 Test

 Interval Notation

2. $x^{2}+4x+4>9$ 3. $2x^{2}+5x\geq 12$

4. $ 9x^{3}-25x^{2}\leq 0$ 5. $(x-3)^{2}<1$

6. $ 3x^{3}+x^{2}-24x>-8$ 7. $2x^{4}-3x^{2}-20 \leq 0$

SOLVING RATIONAL INEQUALITIES

The concepts of critical numbers and test intervals can be extended to rational inequalities.

The value of a rational expression can change sign only at its ***zeros*** (the x-values for which its numerator is zero) and its ***undefined values*** (the x-values for which its denominator is zero).

These two types of numbers make up the **critical numbers** of a rational inequality.

8. $\frac{x-4}{x+5}<4$ 9. $3+ \frac{3}{x-5}>x$

10. $\frac{2x-7}{x-5}\leq 3$ 11. $\frac{x^{2}-2x-15}{x-2}\geq 0$